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Cosmological Probes

Problem Set 3

Hints: 12.10.2023
Due/Solutions: 19.10.2023

Note:

The numerical part of this exercise can be solved on the JupyterHub which you can access through Moodle. Please ensure you have opened the JupyterHub once through moodle before trying to access the assignments. For most problems, we use the cosmology code PyCosmo. To learn more about PyCosmo, you can have a look at the online documentation at `https://cosmo-docs.phys.ethz.ch/PyCosmo/` and read the corresponding paper at `https://arxiv.org/abs/2005.00543`.

1 Age of the Universe

We recall the Friedmann equation, which describes the evolution of the scale factor $a(t)$:

$$H^2(t) = \frac{8\pi G}{3}\left[\rho(t) + \frac{\rho_{\rm crit} - \rho_0}{a^2(t)}\right], \tag{1}$$

where $H(t) \equiv \dot{a}/a$ is the Hubble rate, G is Newton's constant, $\rho(t)$ is the energy density in the unverse as a function of time with ρ_0 being its value today. The critical density $\rho_{\rm crit} \equiv \frac{3H_0^2}{8\pi G}$.

1. Assume that the Universe is flat with matter and a cosmological constant, whose energy density remains constant with time. Re-write the Friedmann equation as

$$\mathrm{d}t = H_0^{-1}\frac{\mathrm{d}a}{a}\left[\Omega_\Lambda + \frac{1-\Omega_\Lambda}{a^2}\right]^{-1/2}, \tag{2}$$

   where Ω_Λ is the ratio of the energy density in the cosmological constant to the critical density.

2. We can integrate this equation from $a = 0$ (when $t = 0$) until today ($a = 1$) to get the age of the universe today. Compute the integral for the following cases:

   (a) a universe with only matter $\Omega_\Lambda = 0$ (analytically).

   (b) a universe dominated by dark energy $\Omega_\Lambda = 0.7$ (numerically).

   (c) For a fixed H_0, which universe is older?

2 Angular diameter distance

Consider a galaxy of physical (visible) size of 5 kpc (1 pc $\approx$ 3.26 light-years). What angle would the galaxy subtend if situated at redshift 0.1? Redshift 1.0?

1. Do the calculation analytically in a flat universe, that contains only matter with $\Omega_M = 1.0$.
2. Use PyCosmo and do the calculation numercally for a universe with the following parameters $\Omega_M = 0.25$, $\Omega_\Lambda = 0.7$ and $\Omega_b = 0.05$.